# NAG Toolbox for MATLAB

# e02af

## 1 Purpose

e02af computes the coefficients of a polynomial, in its Chebyshev-series form, which interpolates (passes exactly through) data at a special set of points. Least-squares polynomial approximations can also be obtained.

## 2 Syntax

```
[a, ifail] = e02af(f, 'nplus1', nplus1)
```

## 3 Description

e02af computes the coefficients $a_j$, for $j = 1, 2, \ldots, n + 1$, in the Chebyshev-series

$$\tfrac{1}{2}a_1 T_0(\bar{x}) + a_2 T_1(\bar{x}) + a_3 T_2(\bar{x}) + \cdots + a_{n+1} T_n(\bar{x}),$$

which interpolates the data $f_r$ at the points

$$\bar{x}_r = \cos((r-1)\pi/n), \qquad r = 1, 2, \ldots, n+1.$$

Here $T_j(\bar{x})$ denotes the Chebyshev polynomial of the first kind of degree $j$ with argument $\bar{x}$. The use of these points minimizes the risk of unwanted fluctuations in the polynomial and is recommended when the data abscissae can be chosen by you, e.g., when the data is given as a graph. For further advantages of this choice of points, see Clenshaw 1962.

In terms of your original variables, $x$ say, the values of $x$ at which the data $f_r$ are to be provided are

$$x_r = \tfrac{1}{2}(x_{\max} - x_{\min})\cos(\pi(r-1)/n) + \tfrac{1}{2}(x_{\max} + x_{\min}), \qquad r = 1, 2, \ldots, n+1$$

where $x_{\max}$ and $x_{\min}$ are respectively the upper and lower ends of the range of $x$ over which you wish to interpolate.

Truncation of the resulting series after the term involving $a_{i+1}$, say, yields a least-squares approximation to the data. This approximation, $p(\bar{x})$, say, is the polynomial of degree $i$ which minimizes

$$\tfrac{1}{2}\epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2 + \cdots + \epsilon_n^2 + \tfrac{1}{2}\epsilon_{n+1}^2,$$

where the residual $\epsilon_r = p(\bar{x}_r) - f_r$, for $r = 1, 2, \ldots, n+1$.

The method employed is based on the application of the three-term recurrence relation due to Clenshaw 1955 for the evaluation of the defining expression for the Chebyshev coefficients (see, for example, Clenshaw 1962). The modifications to this recurrence relation suggested by Reinsch and Gentleman (see Gentleman 1969) are used to give greater numerical stability.

For further details of the algorithm and its use see Cox 1974 and Cox and Hayes 1973.

Subsequent evaluation of the computed polynomial, perhaps truncated after an appropriate number of terms, should be carried out using e02ae.

## 4 References

Clenshaw C W 1955 A note on the summation of Chebyshev series *Math. Tables Aids Comput.* **9** 118–120

Clenshaw C W 1962 Mathematical tables *Chebyshev Series for Mathematical Functions* HMSO

Cox M G 1974 A data-fitting package for the non-specialist user *Software for Numerical Mathematics* (ed D J Evans) Academic Press

Cox M G and Hayes J G 1973 Curve fitting: a guide and suite of algorithms for the non-specialist user *NPL Report NAC26* National Physical Laboratory

Gentleman W M 1969 An error analysis of Goertzel's (Watt's) method for computing Fourier coefficients *Comput. J.* **12** 160–165

# 5 Parameters

## 5.1 Compulsory Input Parameters

1: **f(nplus1) – double array**

For $r = 1, 2, \ldots, n+1$, $\mathbf{f}(r)$ must contain $f_r$ the value of the dependent variable (ordinate) corresponding to the value

$$\bar{x}_r = \cos(\pi(r-1)/n)$$

of the independent variable (abscissa) $\bar{x}$, or equivalently to the value

$$x(r) = \tfrac{1}{2}(x_{\max} - x_{\min})\cos(\pi(r-1)/n) + \tfrac{1}{2}(x_{\max} + x_{\min})$$

of your original variable $x$. Here $x_{\max}$ and $x_{\min}$ are respectively the upper and lower ends of the range over which you wish to interpolate.

## 5.2 Optional Input Parameters

1: **nplus1 – int32 scalar**

the number $n+1$ of data points (one greater than the degree $n$ of the interpolating polynomial).

*Constraint*: **nplus1** $\geq 2$.

## 5.3 Input Parameters Omitted from the MATLAB Interface

None.

## 5.4 Output Parameters

1: **a(nplus1) – double array**

$\mathbf{a}(j)$ is the coefficient $a_j$ in the interpolating polynomial, for $j = 1, 2, \ldots, n+1$.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

# 6 Error Indicators and Warnings

Errors or warnings detected by the function:

**ifail** $= 1$

On entry, **nplus1** $< 2$.

# 7 Accuracy

The rounding errors committed are such that the computed coefficients are exact for a slightly perturbed set of ordinates $f_r + \delta f_r$. The ratio of the sum of the absolute values of the $\delta f_r$ to the sum of the absolute values of the $f_r$ is less than a small multiple of $(n+1)\epsilon$, where $\epsilon$ is the ***machine precision***.

# 8 Further Comments

The time taken is approximately proportional to $(n+1)^2 + 30$.

For choice of degree when using the function for least-squares approximation, see Section 3.2 in the E02 Chapter Introduction.

## 9 Example

```
f = [2.7182;
     2.5884;
     2.2456;
     1.7999;
     1.362;
     1;
     0.7341;
     0.5555;
     0.4452;
     0.3863;
     0.3678];
[a, ifail] = e02af(f)
```

```
a =
    2.5320
    1.1303
    0.2715
    0.0443
    0.0055
    0.0005
    0.0000
   -0.0000
   -0.0000
    0.0000
   -0.0000
ifail =
           0
```